Simulation Models for Dynamic Analysis of Machinery

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Abstract This paper presents some aspects of advanced computational methods for dynamic modelling and simulation of machine systems during operation. Approaches based on symbolic dynamics utilising modified Kane's method, Assur-Artobolevsky method, Newton-Euler method, and Lagrange's method, as well as standard systems of closed loop vector equations are discussed in terms of the engineering requirements for dynamic modelling of a range of complex mechanisms. Basic groups of mechanisms including linkages, cam-follower mechanisms and gear-trains have been used to model and simulate various industrial machinery, such as cam-operated mechanisms of a barb-wire machine, cleaning-house rocker drives for preparation of steel rods, driver for left ventricular assist device and a scissors type load lifting device. The main objective of this paper is to demonstrate the key elements of the applied modelling and simulation techniques and to provide a comparative study of the obtained results in terms of the requirements and specific characteristics of different industrial applications. Various aspects of rigid-body and elastic-body analysis are discussed in conjunction with the developed dynamic models. Procedures for position, velocity, acceleration and dynamic force analysis are developed in different computer environments using the interactive symbolic dynamics approach as well as commercial software with advanced graphic interfaces. Animation of the developed machine models is performed under actual operating conditions in order to visualise the designed functions and optimise the desired machine performance with respect to the design constraints.

1. INTRODUCTION

The process of machinery design relies heavily on visualisation and dynamic analysis of the principal functions undertaken during operation for the purpose of producing some useful work. Elements of machines designed to generate specific functions through predetermined motion are called mechanisms. Different types of mechanisms, including linkages, cam-follower mechanisms and gear trains, display different types of motion with distinctively different dynamic characteristics. The configuration of a machine depends on the desired operating functions and on the consisting subassemblies of mechanisms designed to generate those functions. Traditionally, function analysis is done in industry by developing physical models and prototypes of principal mechanisms in machinery. Due to a growing need to reduce the time and cost involved in developing new machinery, aspects of dynamic modelling and simulation of machinery operation within a computer environment have gained increasing importance in recent years. More specifically, physical prototypes of mechanisms and machines are being replaced in the design stage by matching computer-based prototypes capable of simulating actual motion and dynamic features in real-time. Recent developments in information technology have had a profound effect on dynamic analysis, enabling efficient formulation and integration of equations of motion in conjunction with computerised symbol manipulation.

The main dynamic modelling techniques which can be used for analysis of mechanical mechanisms (as subsets of more complex machine systems) and other articulated multibody systems in general, are the Newton-Euler method, Lagrange's method and Kane's method. Newton-Euler and Lagrange are more traditional techniques, whereas Kane's method is a contemporary technique, which is gaining increasing popularity due to its particular suitability for computer-based implementation. Kane's method utilises a vector-based construction, which leads to a more computationally efficient linearised system of equations of motion. Unlike the Newton-Euler method which is based on an iterative approach and Lagrange's method which results in a non-linear system of differential equations, Kane's method is based on the linearisation of generalised velocities, which results in a linear system of differential equations (Kane and Levinson, 1985). Generalised velocities compliment the notion of generalised coordinates in a more useful way than the traditional generalised velocities of Lagrangian mechanics (Mingori, 1995). Additionally, Kane's equations allow the generalised forces to be determined without the need to invoke the concept of virtual work, which is of particular importance when, generalised active and inertia forces are included in the formulation of the problem (Kane, 1986).

Due to the complexity of real machine systems it is often useful to adopt an inverse dynamics approach by modifying and combining existing analytical models with experimental data (Amirouche, 1992). Also, while in some cases machine elements can be considered rigid thus justifying a rigid-body analysis approach, in other cases they are so elastic that an elastic-body dynamic analysis must be used. Problems associated with the approximation of machine elements by rigid-body members in dynamic analysis are encountered in practice through increased vibration and noise levels, wear and accelerated deterioration of machine members in general, and in some cases through fatigue failure in critical cross-sections (Shigley and Uicker, 1995). This is true in particular in case of high operational speeds and loads, high length to diameter ratios of machine elements, etc. Because of this, dynamic modelling and simulation of machinery involves in large extent engineering judgment and intuition, especially when there is insufficient information about design and operational characteristics of the machine.

The work presented in this paper draws on a number of research and development projects undertaken by the author in collaboration with industry. Central to all projects is the application of computerised symbolic dynamics and motion analysis techniques. Given examples include simulation models of industrial machinery, such as cleaning-house rocker drives for preparation of steel rods, cam-operated mechanisms of a barb-wire machine, driver for left ventricular assist device and a scissors type load lifting device. All models are developed using different features of computer software Maple and Pro/Mechanica (Applied Motion). The presented methodology emphasises, in particular, animation and the process of matching the dynamic characteristics of individual components and subassemblies in motion with the desired output function of the entire machine.

2. BRIEF THEORETICAL BACKGROUND

All mechanisms presented here are modelled using both interactive symbolic dynamics in Maple (which is also the symbolic engine of software Matlab) and an advanced rigid-body dynamics module of software Pro/Mechanica. Common to both approaches is the application of Kane's method for the formulation of system's equations of motion. Depending on the type and configuration of the mechanism, different mathematical modelling procedures have been used in the analysis.

2.1 Formulation of Equations of Motion

Kane's equations for an unconstrained multibody dynamic system, modelled using n generalised coordinates, are represented by:

$$f_l + f_l^* = 0 \qquad l = 1,\ldots,n \tag{1}$$

where fl and fl^* denote the generalised active and inertia forces respectively (Amirouche, 1992). Also,

$$f = W(q,t)u + X(q,t) \tag{2}$$

$$\dot{q} = Wu + X \tag{3}$$

where W and X represent a linear approximation of f to the generalised velocities u, q is the first derivative of generalised coordinates q (Mingori, 1995).

The linearisation of the generalised velocity results in a system of linear differential equations, which describe the dynamic behaviour of the system. Due to the fact that Kane's equations are easily described in a matrix form, computer manipulation of the terms is simplified. The matrix form of the system's equations of motion based on Kane's method is given below,

$$[a]\{\ddot{x}\} + [b]\{\dot{x}\} + [c]\{\dot{x}\} = \{f\} \tag{4}$$

where a is the generalised mass matrix, b and c are the non-linear velocity (b and c are also known as the dynamic damping of the system), f denotes the generalised external forces encompassing resultant external force and moment acting on the body, $\dot{x}$ is the first derivative and $\ddot{x}$ is the second derivative of the generalised coordinates.

The resultant system of equations of motion is suitable for use in conjunction with experimental data (kinematic or dynamic) through an inverse dynamics approach. The inverse (or forward) dynamics approach to mechanism analysis is a technique that combines theoretical and experimental aspects of dynamic modelling. Once a mathematical model of the mechanism is formulated, it is then possible to input experimentally determined data into equations of motion, solve the resulting system of equations and determine (or validate) the dynamic behaviour of the model.

2.2 Geometrical Description of Typical Mechanisms

Mechanisms used in industrial machinery are usually of complex geometry. Therefore, prior to describing the dynamics of the entire mechanism by formulating system's equations of motion it is of paramount importance to describe precisely the geometry of key elements. This is particularly true in case of cam-follower mechanisms where the motion of the follower is contained in the shape of the cam. In such mechanisms, it is necessary to mathematically describe the curvature of the cam, as this is usually an input to the corresponding dynamic system.

Cam-follower mechanisms represent one of the most useful function generators in industrial machinery. It is possible to specify virtually any output function by creating an appropriate curved surface on the cam to generate the desired function in the motion of the follower. Cam functions over the entire motion interval of interest is typically a piecewise function which can be approximated using different geometrical entities such as those based on parabolic motion, simple harmonic motion, cycloidal motion or polynomial motion characteristics (Mabie and Reinholtz, 1987). In all cases, the creation of smooth cam profiles without discontinuities in velocity, acceleration, and higher derivatives is critical to the satisfactory operation of all cams. Also, due to the fact that dynamic force is proportional to acceleration, it is generally important to minimise dynamic forces by reducing the magnitude of the acceleration function as well as to keep it continuous. In addition, because kinetic energy is proportional to velocity squared it is important to minimise stored kinetic energy by carefully modelling the velocity functions. All these considerations are directly associated with the geometry of the cam.

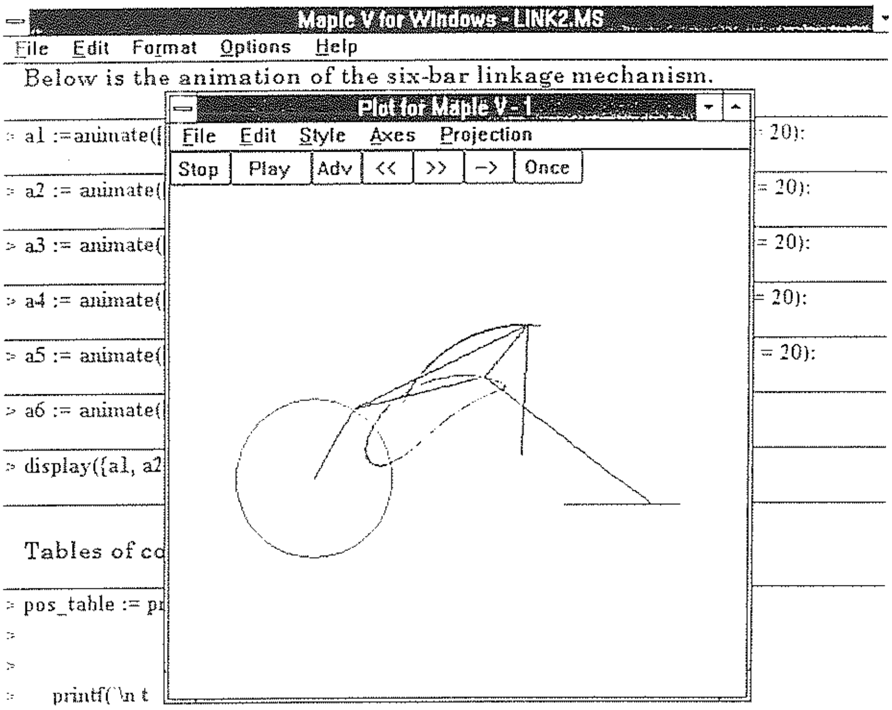
Polynomial functions are the most versatile in cam design due to the fact that they can be tailored to almost any design specification (Norton, 1992). The general form of a polynomial function is,

$$s = C_o + C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n \tag{5}$$

where s is path of the cam and in fact follower displacement, x is the independent variable (either θ/β or time t) and constant coefficients C are the unknowns which when solved generate a particular polynomial equation to suit desired design specification. The degree of the polynomial will depend on the specified design requirements, desired accuracy and machinability of the cam curvature. Cam-follower mechanisms presented in this paper are based on the polynomial description of cam geometry.

Linkage mechanisms have completely different geometrical features. Depending on the design constraints and the desired output functions these mechanisms can have a vast amount of variations in configuration with different number of bars. The complexity of the linkage mechanism increases with the number of bars. For example, while four-bar linkages can be described effectively by directly applying the closed-loop vector equations using a complex number notation in conjunction with Euler's identity, linkage mechanisms with higher number of bars require more advanced modelling techniques.

The six-bar linkage mechanism shown in **Figure 1** and **Figure 2** (which actually represents a sewing machine mechanism) is modelled using the Assur-Artobolevsky method. This method is based on the concept of structural decomposition of the mechanism on principal kinematic pairs. Vector equations are then subsequently developed for each kinematic pair, and position functions for each joint of the mechanism expressed. First and second derivation of these position functions yield velocity and acceleration functions. Once the acceleration function of the mechanism is determined, using the masses of the linkages it is possible to calculate dynamic forces. Analytical expressions for position, velocity and acceleration are particularly important because they enable full animation of the mechanism in real-time.




```
> Position_Animation:=proc()
> LinkAC:=animate([XA+d*(XC-XA),YA+d*(YC-YA),d=0..1],
> t=0..0.11,axes=none,color=black,frames=25,numpoints=2):
> LinkBC:=animate([XB+d*(XC-XB),YB+d*(YC-YB),d=0..1],
> t=0..0.11,axes=none,color=black,frames=25,numpoints=2):
> LinkAB:=animate([XA+d*(XB-XA),YA+d*(YB-YA),d=0..1],
> t=0..0.11,axes=none,color=black,frames=25,numpoints=2):
> LinkBE:=animate([XB+d*(XE-XB),YB+d*(YE-YB),d=0..1],
> t=0..0.11,axes=none,color=black,frames=25,numpoints=2):
> LinkCD:=animate([XC+d*(XD-XC),YC+d*(YD-YC),d=0..1],
> t=0..0.11,axes=none,color=black,frames=25,numpoints=2):
> LinkOA:=animate([(XA)*d,(YA)*d,d=0..1],
> t=0..0.11,axes=none,color=black,frames=25,numpoints=2):
> PathA:=animate([XA,YA,t=0..0.12],d=0..1,
> color=blue,axes=NONE,frames=25):
> PathB:=animate([XB,YB,t=0..0.12],d=0..1,
> color=violet,axes=NONE,frames=25):
> PathC:=animate([XC,YC,t=0..0.12],d=0..1,
> color=red,axes=NONE,frames=25):
> PathD:=animate([XD,YD,t=0..0.12],d=0..1,
> color=brown,axes=NONE,frames=25,title='Position_Animation'):
> display({LinkOA,LinkAB,LinkAC,LinkBC,LinkCD,LinkBE,PathA,PathB,PathC,PathD});
```

Figure 1. Animation of the six-bar linkage mechanism using software Maple

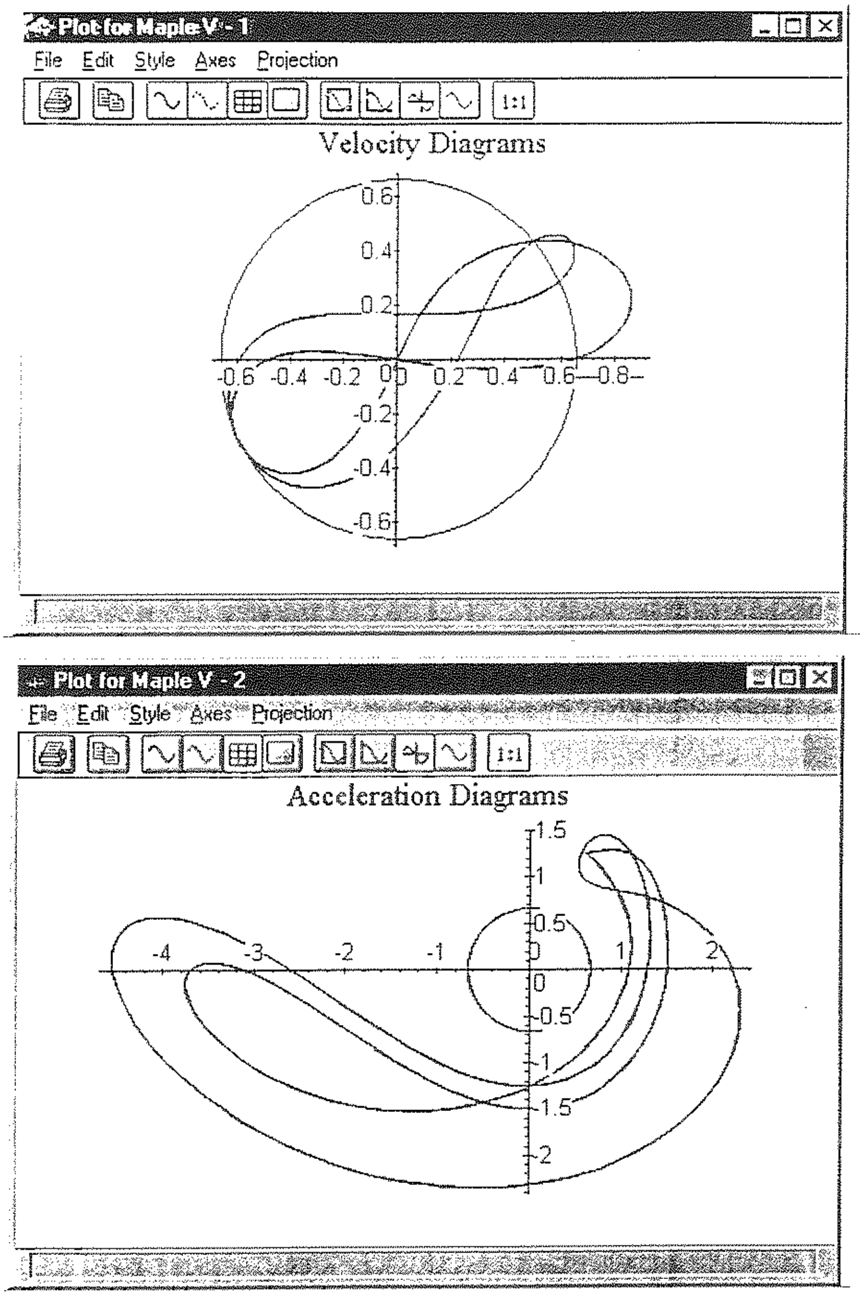


Figure 2. Simulation of velocity and acceleration diagrams for six-bar linkage mechanism using Maple

3. CASE STUDIES

Various aspects of the dynamic modelling approaches described in previous sections have been implemented in a variety of research and development projects in collaboration with industry. Case studies presented in this paper include some selected examples of industrial machinery, such as cleaning-house rocker drives for preparation of steel rods, driver for left ventricular assist device and a scissors type load lifting device. Also, characteristic results obtained through modelling and simulation of principle mechanisms during operation are discussed in conjunction with the developed dynamic models. Animation of the developed machine models is performed under actual operating conditions in order to visualise the designed functions and optimise the desired machine performance with respect to the design constraints.

3.1 Cleaning-House Rocker Drives

The cleaning-house consists of five sulfuric acid baths that are used for immersion of coils of rod. Simple stationary immersion is not adequate because the sulfuric acid does not penetrate the material sufficiently, localised low concentration of acid is formed and scale removal is poor. Because of this, a rocker mechanism is used to agitate the coils of rod gently to enhance scale removal. Over the years in the existing rocker mechanism various parts have deteriorated and due to periodical maintenance, repairs and replacements of assembly parts, the driving cranks of the mechanism have become synchronised resulting in higher peak loads. This has a number of undesirable effects on the machine system, such as unbalance and overload of the motor, reduction of efficiency and irregular agitation of the coils of rod resulting in poor quality of the product.

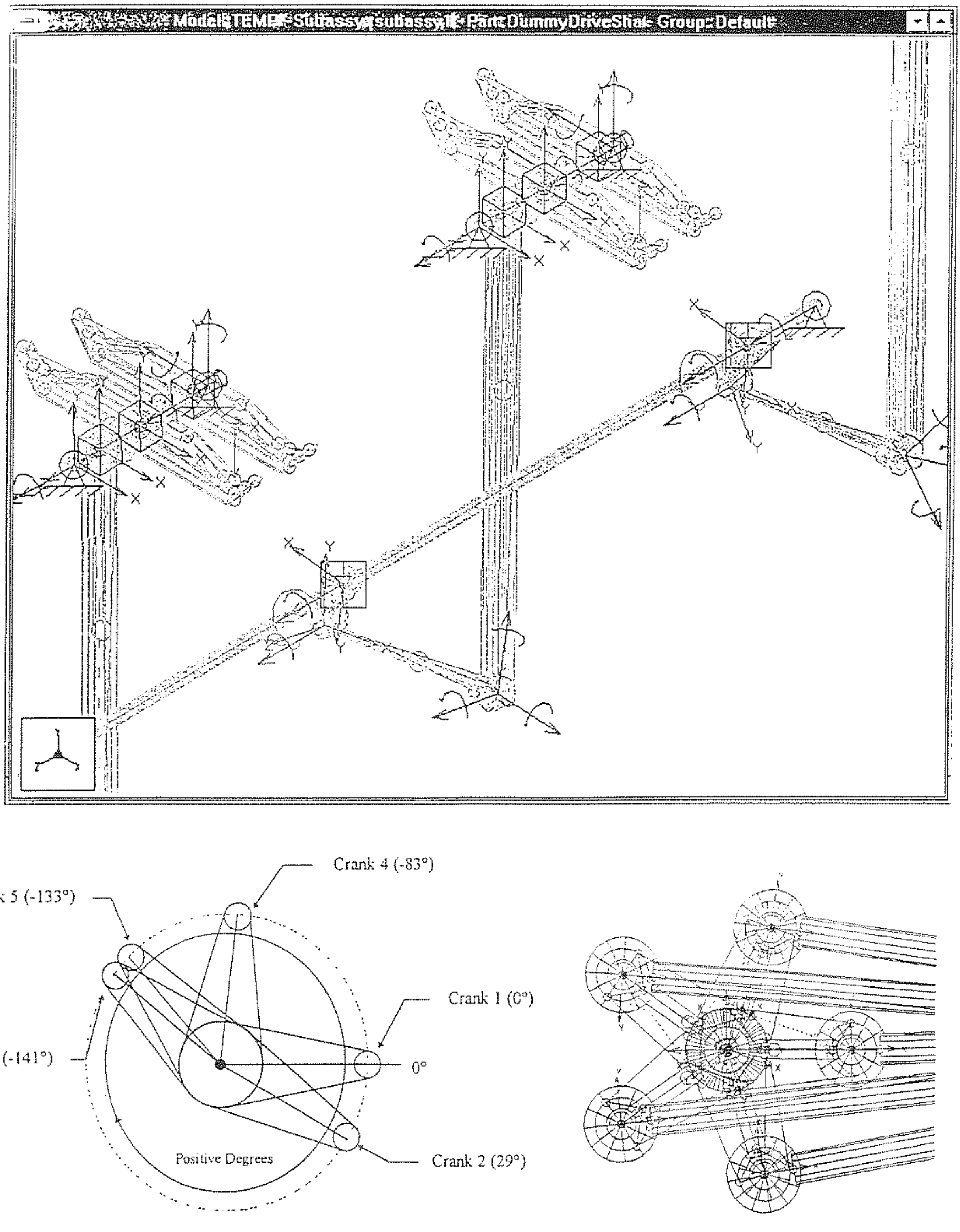
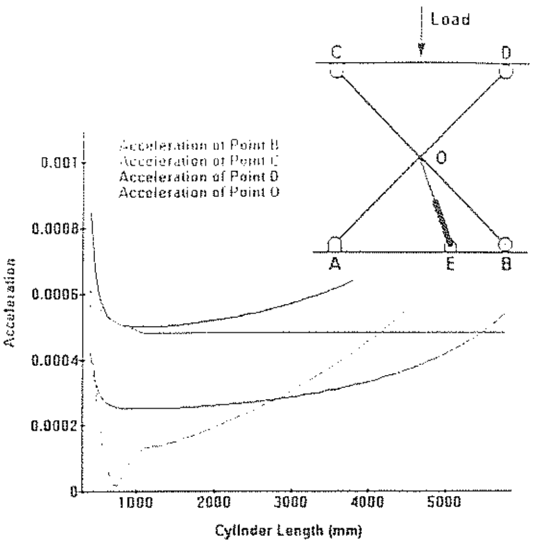


Figure 3. Computer model of cleaning-house rocker drives with optimised crank positions

This mechanism was modelled using the symbolic dynamics approach in Maple and motion analysis features of software Pro/Mechanica. In this process, of particular importance was the optimum correlation of the crank angles with respect to each other for minimum loading of the motor during normal operation. **Figure 3** shows the computer model of the rocker drive mechanism developed using software Pro/Mechanica with initial values and optimised values of crank angles. A general out-of-balance loading arrangement was simulated for all five baths and an optimal crank separation of 72° determined. For this loading condition the torque required at the secondary shaft was minimised to 14 Nm. This would alleviate the current overload of the motor. Position, velocity and acceleration analysis of this mechanism was also carried out using Maple in order to validate the simulation results.

3.2 Hydraulic Lifting Platform

A similar approach was applied in modelling the linkage mechanism of a hydraulic lifting platform. Development of position, velocity and acceleration functions in parametric form using symbolic dynamics enabled animation of various design concepts and optimisation of the location of actuation via the hydraulic cylinder. Some characteristic results of this simulation are shown in **Figure 4**.




```
Simulation :=

  proc(AEO,ABC,theta2_min,theta2_max,AO)
     with(plots);
     if nargs = 5 and type(AEO,numeric) and type(ABC,numeric) and
        type(theta2_min,numeric) and type(theta2_max,numeric) and
         then
        aeo := convert(AEO*degrees,radians);
        abc := convert(ABC*degrees,radians);
        t2_min := convert(theta2_min*degrees,radians);
        t2_max := convert(theta2_max*degrees,radians);
        AE := evalf(sin(Pi-aeo-abc)*AO/sin(aeo));
        min_cyl := evalf(sqrt(AO^2+AE^2-2*AO*AE*cos(t2_min)));
        max_cyl := evalf(sqrt(AO^2+AE^2-2*AO*AE*cos(t2_max)));
        ad := animate([-u*(EO^2-AO^2-AE^2)/AE,
            2*AO*u*sin(arccos(-1/2*(EO^2-AO^2-AE^2)/AO/AE)),u = 0
            EO = min_cyl .. max_cyl,frames = 10,axes = none,color
        bc := animate([-u*(EO^2-AO^2-AE^2)/AE,
            (2-2*u)*AO*sin(arccos(-1/2*(EO^2-AO^2-AE^2)/AO/AE)),u
            EO = min_cyl .. max_cyl,frames = 10,axes = none,color
        cd := animate(
            [2*AO*u,2*AO*sin(arccos(-1/2*(EO^2-AO^2-AE^2)/AO/AE)),
            EO = min_cyl .. max_cyl,frames = 10,axes = none,color
        ab := animate([2*AO*u,0],u = 0 .. 1],EO = min_cyl .. max_cy
            axes = none,color = blue);
        eo := animate([-1/2*u*(EO^2-AO^2-AE^2)/AE+AE-AE*u,
            AO*u*sin(arccos(-1/2*(EO^2-AO^2-AE^2)/AO/AE)),u = 0 ..
            EO = min_cyl .. max_cyl,frames = 10,axes = none,color
        display({cd,ab,eo,ad,bc},title = `Cylinder Force`)
     else ERROR(`The entered arguments are not of the correct numbe
     fi
  end
```

Figure 4. Simulation of a hydraulic lifting platform

3.3 Cam-Operated Driver for LVAD

Left Ventricular Assist Device (LVAD) is an artificial heart, which supports circulation of the left side (ventricle) of the heart. It may be implemented (internal) or extracorporeal (external). The LVAD is used to pump blood in parallel with a heart that is weakened by disease or post-surgical trauma. One of the most commonly used types of LVAD is the diaphragm type, pneumatically driven, pulsatile blood pump. In practice, there is a demand that the blood pump and the mechanical driver operate under various conditions. This case study shows a glimpse of an ongoing research project which aims to develop a dynamic model of the complete cam-driven mechanism to define input variables for the given output requirements and vice versa, to define the system response for the given set of input variables. Central to this work is the design of cam curvatures using appropriate polynomial functions for different operating conditions and needs. This was also achieved using the symbolic dynamics approach.

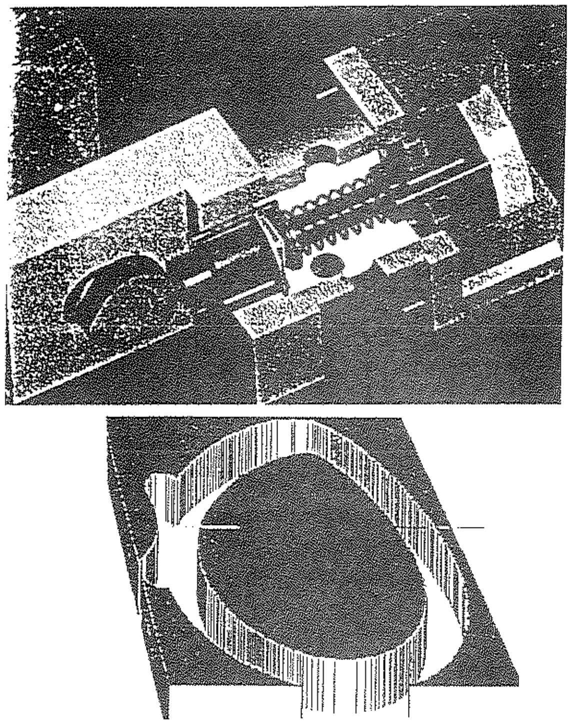


Figure 5. Computer model of driver for LVAD

4. CONCLUSION

The paper presented a number of case studies, which aimed at illustrating main aspects of developed simulation models of industrial machinery. With this in mind, the main objective was to outline the contemporary approaches and techniques applied for dynamic modelling and simulation, while detailed discussion of the underpinning mathematical models for each separate case does not fall in the scope of this paper. Output functions of all mechanisms presented in this paper were developed in parametric form for detailed design studies using symbolic dynamics. For example, the simulation model of the cleaning-house rocker drive highlighted the optimal crank positions, but more important than that was the motion study in a computer environment that helped minimise acceleration and operating loads.

5. REFERENCES

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